## CS109 Section 1

## Problem 1

A student is taking the computer-based GRE test. As the student answers questions, the test adapts its level of difficulty to their abilities. Let's say that the student is in the math section, and the test is trying to estimate whether or not they know geometry based on whether they answer the first geometry question correctly. The test's prior belief that the student knows geometry is q. The test also knows the following: Given that a student knows geometry, the probability that they solve the first question correctly is  $p_1$ . Additionally, given that a student does not know geometry, the probability that the solve the first questions correctly is  $p_2$ . Given that the student solves the first question correctly, what is the probability that they know geometry?

## Problem 2

Continuing the story from Problem 1: Now the test is trying to estimate what level of experience a student has with geometry. There are 4 levels: Beginner, Intermediate, Expert, and Euclid, and each student is in exactly 1 level. The test's prior belief that a student is in a certain level is as follows: Beginner is  $b_1$ , Intermediate is  $b_2$ , Expert is  $b_3$ , and Euclid is  $b_4$ . The conditional probability that a student solves the first question correctly given their level is as follows: Beginner is  $d_1$ , Intermediate is  $d_2$ , Expert is  $d_3$ , and Euclid is  $d_4$ . Find the conditional probability that a student is in a certain level given that they solve the first question correctly. Do this for each of the 4 levels.